Progressing through the van Hiele levels with Cabri-géomètre

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The influence of the dynamic geometry tool, Cabri-géomètre, on the learning of geometry was investigated. Twelve Year 7/8 pupils were divided into two groups on the basis of van Hiele pre-tests: Group I, pupils at Level 0 or 1, and Group II, the remaining pupils. Both groups completed six lessons with structured Cabri worksheets then Group II pupils completed several construction tasks, documented by means of Cabri files and tape-recorded conversations. All pupils increased their van Hiele levels for some or all of the concepts involved.

Cabri-géomètre™ is one of several dynamic geometry software tools which have created much interest amongst mathematics educators. Accurately measured constructions can be produced quickly, enabling explorations which previously would have been seen as too tedious and time-consuming. Use of these computer tools also overcomes the limitation of students' lack of precision which may result in invalid interpretations when working with pencil and paper. More importantly, continuous transformations of drawings are possible in the dynamic environment, allowing pupils to see relationships which otherwise may not have been apparent.

The van Hiele theory of geometry learning

Children's learning of geometry was investigated in the 1950s by the Dutch researchers, Pierre van Hiele (1957/1984) and his wife, Dina van Hiele-Geldof (1957/1984). Since then, their theory of levels of learning has been the subject of research in many countries. According to the theory, there is a hierarchy of levels through which children progress:

- Level 1 Shapes are recognised by their visual appearance alone.
- Level 2 One or more properties of a geometric shape are recognised.
- Level 3 Relationships between properties are recognised.

Two further levels were identified by the van Hieles, but Level 4 would be reached by only a few pupils and Level 5 normally would not be relevant to secondary school geometry. Dina van-Hiele Geldof noted that it took 20 lessons to take her 12 year-old pupils from Level 1 to Level 2 and a further 50 lessons to take them from Level 2 to Level 3.

Aims of the study

The study was designed to investigate the following hypotheses:

1. Learning geometry with Cabri allows students to progress from one van Hiele level to the next more rapidly than suggested by Dina van Hiele-Geldof.
2. When using the dynamic environment of Cabri, students initially regress in the thinking displayed in their constructions to a level lower than their conventionally measured van Hiele level.
3. Learning with Cabri contributes to the understanding and use of correct geometric language.

The participants

The study was undertaken in a private girls' school in Melbourne with 12 members of an accelerated Year 7/8 class (ages 11-12). They had a wide variety of backgrounds at Grade 6 level and a wider than expected range of abilities for an accelerated class, although all were highly motivated. The girls had their own notebook computers, but had not used Cabri prior to this study.

Methodology

The study was essentially a case study. It was both quantitative, involving measurement of changes in van Hiele levels associated with the use of Cabri, and qualitative, documenting and analysing pupils' geometric language and methods of construction of geometric figures with Cabri. Data collection, in the form of students' test responses, Cabri files and taped conversations, took place over a period of four weeks. The methodology involved the following stages:

1. Geometric terms test
2. Van Hiele pre-tests
3. Cabri worksheet lessons
4. Cabri constructions for selected students
5. Van Hiele post-test
6. Further Cabri constructions for selected students

Geometric terms test: Pupils matched terms such as parallel, perpendicular, isosceles triangle, scalene triangle with appropriate drawings.

Van Hiele test: Written test developed by Lawrie (personal communication, 1997) from Mayberry's van Hiele test (1983), testing the concepts squares, right-angled triangles, parallel lines and isosceles triangles. On the basis of the van Hiele pre-test, the twelve pupils were divided into two groups:

Group I: Six pupils who were at Level 0 or 1 on all or most concepts.

Group II: Six pupils who were at Level 2 or 3 on all or most concepts.

The Cabri lessons (All lessons were 45 minutes and were spread over three weeks.)

Lesson 1: Cabri introduction. Students explored the menu options.

Lesson 2: Cabri exploration. Students were directed to investigate the difference in behaviour between basic point, point on object and intersection points. They also marked and measured angles, measured line segments, labelled points.

Lessons 3-8: *Learning with Cabri.* Students completed a set of structured activities which served the dual purpose of allowing them to become more confident in using Cabri while at the same time constructing or reinforcing their own knowledge of some basic properties of parallel lines, triangles and quadrilaterals. Teacher intervention while the students were using Cabri was minimal, the most frequent reason being to assist them when they had chosen an inappropriate Cabri point. Approximately 5 minutes were spent in class discussion at the end of each lesson reinforcing the properties the students had explored. These explorations were:

1. Angles in a straight line and vertically opposite angles
2. Angles in a pair of parallel lines cut by a transverse line
3. Angle sum of triangles and external angles of triangles
4. Angle sum of quadrilaterals and diagonals of quadrilaterals

Construction tasks: On completion of the Cabri worksheet activities, Group II students were required to construct the following drag-resistant figures in Cabri: right-angled triangle, rectangle, parallelogram, isosceles triangle. These tasks took one or two lessons. The pupils' construction steps were retraced in the saved files by means of Cabri's *History* option. Analysis of the constructions enabled the author to identify the van Hiele levels of thinking displayed in the Cabri constructions.
Van Hiele post-test: The Mayberry/Lawrie van Hiele test was administered as a post-test. (The students had not been told their scores on the pre-test.)
Further construction tasks: Following the Mayberry/Lawrie post-test, selected Group II students constructed letter A and 'House' shapes (Figure 1) in Cabri so that the shapes would retain their basic properties when dragged.

Figure 1. Letter A and House construction tasks

Results

Table 1 shows the pre-test and post-test levels for the four concepts for Group I students. All six students increased by one level in at least two of the four concepts and 15 of the total of 24 individual concept levels increased. All six students were now at Level 2 (Students C, D, I and K) or 3 (Students B and F) for Squares and only two students remained at Level 1 on each of the other three concepts. Student F had progressed from Level 0 to Level 2 for isosceles triangles. These increases, which occurred after six Cabri lessons (lessons 3-8), contrast with the reported 20 lessons taken by Dina van Hiele-Geldof to bring her 12 year-old pupils from Level 1 to Level 2.

Table 1. Group I pre-test and post-test van Hiele levels for four concepts

Student	Squares		Right-angled triangles		Isosceles triangles		Parallel lines	
	Pre-test	Post-test	Pre-test	Post-test	Pre-test	Post-test	Pre-test	Post-test
B	2	3	1	1	1	1	1	2
C	2	2	1	2	1	2	1	2
D	1	2	1	2	0	1	1	1
F	2	3	1	1	0	2	1	1
I	1	2	1	2	2	2	1	2
K	2	2	2	2	1	2	1	2

Note: Shaded cells indicate where increases in van Hiele level have occurred

Table 2 shows a comparison of pre-test and post-test results for the four concepts for the Group II students. Five of the six students progressed from Level 2 to Level 3 in two of the four concepts, with Students G, J and L progressing to Level 3 on three concepts. In three cases, students satisfied the criteria for Level 3, but not for Level 2, for a particular concept and could not be classified. This may indicate that these students were in transition between levels. Given that Students A, E, G and H were already at Level 3 for one of the four concepts, increases occurred in 14 of the possible 20 individual concept levels. These increases, which occurred after seven or eight Cabri lessons, contrast with the 50 lessons taken by Dina van Hiele-Geldof to bring her 12 year-old pupils from Level 2 to Level 3.

Table 2. Group II pre-test and post-test van Hiele levels for four concepts

Student	Squares		Right-angled triangles		Isosceles triangles		Parallel lines	
	Pre-test	Post-test	Pre-test	Post-test	Pre-test	Post-test	Pre-test	Post-test
A	2	3	NC	NC	2	3	3	3
E	2	3	3	3	NC	3	2	3
G	2	3	NC	3	3	3	2	3
H	2	2	2	3	2	2	3	3
J	2	2	2	3	2	3	2	3
L	2	3	1	3	2	2	2	3

Note: NC Not classified
Shaded cells indicate where increases in van Hiele level have occurred

The van Hiele levels displayed in the students' constructions

All Group II students, with the exception of Student E, used Level 1 by-eye strategies to varying degrees when constructing their first figure. When it came to other shapes, some of the students still attempted by-eye constructions before resorting to the Construction menu, even though they knew from constructing their first shape that by-eye constructions were "messed up" (Healy, Hoelzl, Hoyles and Noss, 1994) by dragging. Others, however, tended to be less dependent on by-eye methods and immediately explored geometric construction methods, confidently selecting the appropriate menu items. It was apparent also that the students' understanding of geometric language was developing through using the construction tools.

The following examples illustrate the progress in thinking which occurred. Student A's first attempt at the construction of a rectangle (Figure 2) was purely visual.

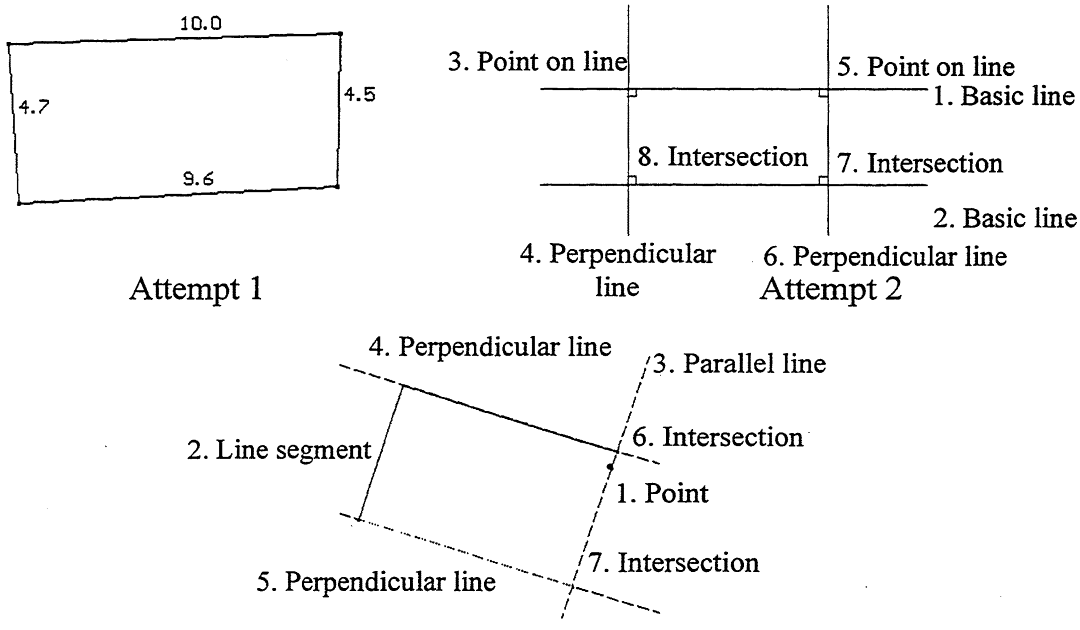

Figure 2. Student A: Construction of a rectangle

Student A's second attempt was partly visual, with two *Basic lines* aligned parallel to the lower edge of the screen, but the vertical lines were constructed using the

Construction menu item, *Perpendicular line*. Her final, successful attempt was based on correct geometric construction using both *Parallel line* and *Perpendicular line* from the Construction menu. Student A no longer aligned her rectangle parallel to the screen edges.

A similar progression from purely visual to geometric construction is apparent in the construction of an isosceles triangle by Student G (Figure 3).

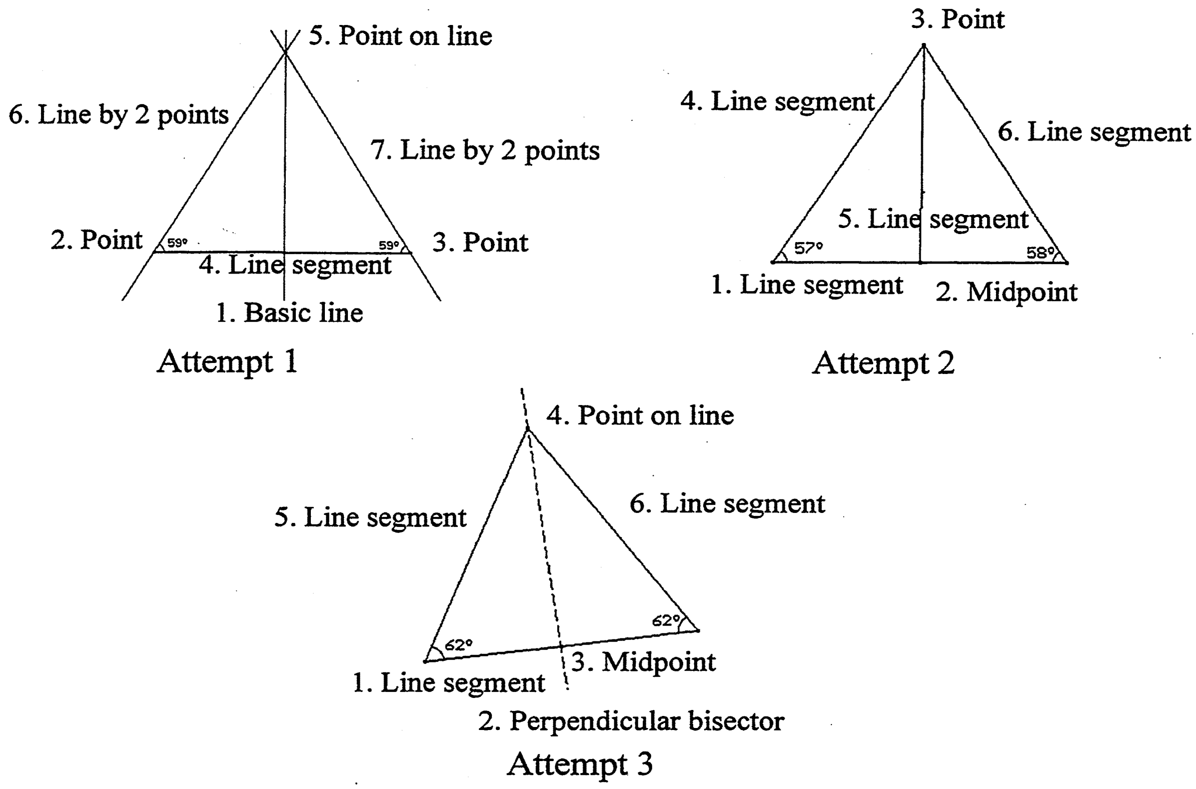

Figure 3. Student G: Construction of an isosceles triangle

In her first attempt, Student G placed Points 2 and 3 by-eye, while in her second attempt, Point 3 was placed by-eye above the geometrically constructed midpoint of the base. In her final construction, which represents a transition to Level 3 thinking, she employed the Construction menu item, *Perpendicular bisector*, a term which she had not known on the Geometric terms pre-test.

The constructions of the letter A and House shapes showed a clear progression from purely visual Level 1 thinking to Level 3 thinking, where the students were giving careful consideration not only to the properties of the figures but to the relationships between the properties. The shapes were mentally dissected and reassembled, allowing the pupils to develop a logical ordering of the construction steps. The initial by-eye attempts provided the visual framework on which to build a geometric construction. Although the students occasionally lapsed back into by-eye methods during these constructions, in general they became extremely focused, confidently selecting the Cabri Creation and Construction tools and using appropriate geometric language. Figure 4 shows the first joint attempt by Students A and G for the House shape, which was partly visual but with geometrically constructed parallel lines.

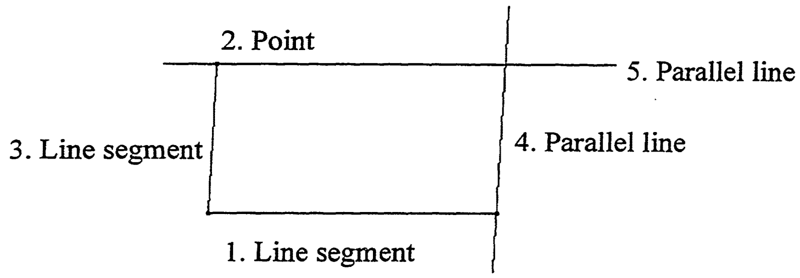

Figure 4. Students A and G: Construction of House shape (Attempt 1)

Distortion of this construction when dragged enabled Students A and G to analyse the shape and think about the relationship between the properties of the component parts, resulting in the Level 3 construction shown in Figure 5.

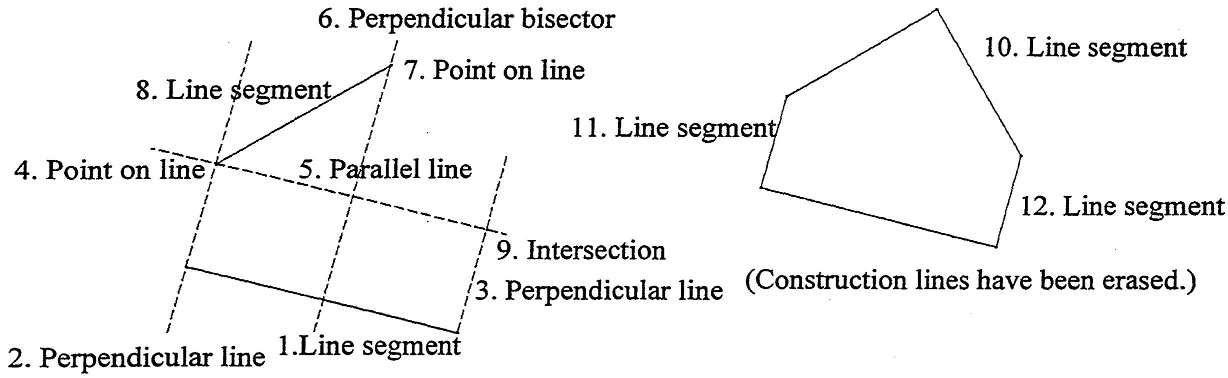

Figure 5. Students A and G: Construction of House shape (Attempt 2)

Validation of constructions

The students all made at least some measurements, either of angles or sides to check their constructions, particularly where they employed by-eye methods. Usually these measurements were made before dragging the figures. As the students left their by-eye constructions behind and used the Construction tools, they generally no longer measured angles or sides and used the drag facility instead as a means of validating their constructions. They seemed to have realised that there was little point in making measurements if their figures were not based on geometric constructions.

Development of geometric language

The use of geometric language by Students A and G was evident throughout their construction of the House shape, and this played an important part in their ability to discuss and verbalise the next step in their construction. As new terms were learned they became incorporated into the students' language. "*Perpendicular thing*" said Student G, only to be corrected by Student A: *"A perpendicular bisector*". The Cabri tools and the students' geometric understanding were becoming intertwined. This confident use of geometric terms supports the van Hieles' belief that knowledge and use of appropriate language is a prerequisite to the development of thinking skills and progression through the levels. Van Hiele-Geldof (1958/1984) notes that "concept and language can be distinguished, but cannot be separated. The thinking operation itself first has to be made conscious through language symbols and the language symbols are a consequence of the thinking operation" (p. 232).

Conclusions

The findings of this study support the hypothesis that using Cabri can indeed enable a more rapid progression through the van Hiele levels than found by van Hiele-Geldof (1957/1984). Pegg (1995, p. 93) notes that students must face a "crisis of thinking" before moving to a higher level. The "messing up" by dragging of Cabri drawings which are not based on valid geometrical constructions, provides students with this crisis of thinking. If they wish to produce a drag-resistant shape they must analyse which properties are to remain invariant and construct their figure accordingly. Students are thus forced into thinking about properties and relationships, facilitating their progression from Level 1 to Level 2 and in turn, from Level 2 to Level 3.

The study indicates that most students do in fact regress temporarily to the security of Level 1 visual construction when using Cabri. All but one of the students applied Level 1 by-eye construction methods, regardless of their initial measured level. Noss and Hoyles (1996) believe that these visual constructions are providing the essential scaffolding on which pupils can base their geometric constructions. While some pupils may be able to operate without by-eye constructions, others seem to require a visual construction before they can identify properties and relationships.

The study highlights the benefits of Cabri in developing pupils' confident use of geometric language, particularly if they are working in pairs where they can discuss their observations and construction strategies. Fundamental to this development, though, is the pupils' freedom to explore the Cabri tools. Once they discover the meaning through exploration of the terms in the menus, they are then able to incorporate these terms into their geometric vocabulary and employ them in their constructions.

Implications for teaching

It is important that Cabri tasks are structured carefully. Pupils first need to be familiar with the Creation items, since these are the tools which allow them to produce their visual shapes. The basic items of points, lines, line segments, triangles and circles can be explored creatively by children with little or no geometry experience. When names of shapes can be matched with a visual image, pupils can be introduced to the measurement of line segments and angles. Pupils at Level 1, such as the Group I students in this study, cope well with structured tasks involving drawing and measuring. Before angles can be measured, however, careful introduction to the various Cabri points is required. Unless pupils are aware of the status of the different points, *Basic point, Point on object, Midpoint* and *Intersection*, frustration and confusion will result.

It is the next stage, the construction of drag-resistant figures, which challenges students to think about properties and relationships. Such activities should be matched to the students' levels to avoid the inevitable confusion and inability to cope which appeared to be occurring when two Group I students, who were just at Level 2, were asked to construct the letter A shape. By gradually increasing the complexity of construction tasks, the teacher can develop the pupils' analytical skills and Level 3 understanding. Exploratory tasks, in which students are encouraged to experiment, conjecture and test and prove their conjectures, tasks which are eminently suited to the Cabri environment, will develop students' understanding to Level 3 and beyond. It is essential, though, that pupils who have just reached a level are given sufficient time to consolidate their understanding at that level.

The role of teacher intervention must also be considered. Hoyles and Noss (1996) noted that intervention involved them in a 'critical tension' due to the need

> to tread the line carefully between the pupils' room for manoeuvre and exploration on the one hand, and our own intentions and structuring on the other. We are convinced that without the former, much of the interesting potentiality of the computer is put at risk. Yet without the latter, our experience suggests that pupils' mathematical learning is at best haphazard (p. 32).

The students in this study required teacher intervention at times to assist them with choosing the appropriate Cabri points, but by carefully structuring the tasks so that they were appropriate to the students' levels, the need for intervention was kept to a minimum.

Any conclusions drawn from this research should take into account that the participants were generally highly motivated, with unrestricted access to the use of computers in Mathematics lessons. The study provides sufficient evidence, however, to justify further research on a larger scale to explore the full potential of dynamic geometry tools such as Cabri in the learning of geometry.

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